

# A Bandstop Filter Constructed in Coupled Nonradiative Dielectric Waveguide

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**Abstract**—The design of a bandstop filter in nonradiative dielectric waveguide is described. The filter makes use of side-coupled guides to realize an equivalent of an open-circuited shunt stub and transmission-line section. Theoretical and practical results of the coupled lines as well as the filter performance are presented.

## I. INTRODUCTION

**I**N ORDER TO realize its promise as a transmission medium, the nonradiative dielectric waveguide (NRD-guide) must support the design of all the necessary active and passive networks for the realization of systems or subsystems. It is important to realize that such networks must be integrated in nature, because the advantages of dielectric waveguides are lost if transitions to and from other guiding media are needed. Up to the present no bandstop filters for integrated use in NRD-guide have been described, and only two bandpass filters [1] have been described.

In this paper, the theoretical frequency response as well as measured transmission response of a third-order bandstop filter is described, and its design discussed. The design is based on the properties of a pair of coupled NRD-guides of which the ports of one line have been short-circuited and side-coupled to the main guide. The theoretical properties of the coupled guides are discussed and compared to practical values.

Both the design and the construction are carried out at *X*-band with a low-dielectric-constant center strip. This limits the useful bandwidth severely, but makes the verification of design principles possible on a limited budget. In practice, use would be made of the high-dielectric-constant materials currently available. The NRD-guide can under such circumstances be used over much increased bandwidths, as discussed in Section VI.

## II. COUPLED GUIDES WITH ARBITRARY PORT CONDITIONS

An equivalent circuit for a pair of coupled NRD-guides with arbitrary port conditions was derived in [2] by making use of signal-flow graphs. Practical figures for coupling were measured; in the case of guides terminated as shown in Fig. 1(a), it was found that a poor correlation exists between the calculated and measured coupling properties.

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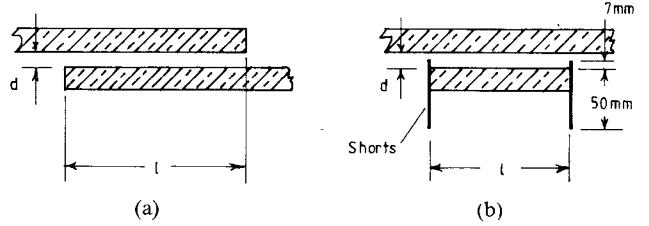


Fig. 1. Coupled NRD-guides. (a) Series coupled. (b) Shunt section.

This is due to the coupling between the unterminated ends of the guides and the adjacent guide. The frequency response obtained was also not suitable for application to practical filters.

In order to reduce the end coupling, the ends of the guides were terminated in short circuits in the form of metal strips extending partly across the gap between the guides, as shown in Fig. 1(b). This structure is very well suited to application as a filter element. If the short circuits are omitted, a very poor response results, with considerable transmission variations in the passband.

The forward transmission coefficient for a pair of guides terminated as in Fig. 1(b) is given by [2]

$$S_{21} = \cos Cl \exp(-j\beta_c l) \left[ 1 - \frac{\sin^2(Cl) \exp(-j2\beta_c l)}{1 - \cos^2(Cl) \exp(-2j\beta_c l)} \right] \quad (1)$$

where the coupling factor is given by

$$C = (\beta_e - \beta_o)/2. \quad (2)$$

The wavenumber in the coupled section is given by

$$\beta_c = (\beta_e + \beta_o)/2 \quad (3)$$

where  $\beta_e$  and  $\beta_o$  are, respectively, the even- and odd-mode wavenumbers and are obtained by solving the transcendental equations [3].

For even modes

$$\tanh(\alpha_y d) = \frac{\tan(\beta_y b) - \frac{\epsilon_r \alpha_y}{\beta_y}}{\frac{\epsilon_r \alpha_y}{\beta_y} \left[ 1 + \frac{\epsilon_r \alpha_y}{\beta_y} \tan(\beta_y b) \right]}. \quad (4)$$

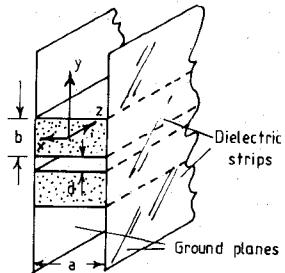


Fig. 2. Defining the cross section of the NRD.

For odd modes

$$\coth(\alpha_y d) = \frac{\tan(\beta_y b) - \frac{\epsilon_r \alpha_y}{\beta_y}}{\frac{\epsilon_r \alpha_y}{\beta_y} \left[ 1 + \frac{\epsilon_r \alpha_y}{\beta_y} \tan(\beta_y b) \right]} \quad (5)$$

For both cases

$$\alpha_y^2 + \beta_y^2 = k_0^2(\epsilon_r - 1) \quad (6)$$

$$\beta^2 = k_0^2 \epsilon_r - (m\pi/a)^2 - \beta_y^2 = k_0^2 - (m\pi/a)^2 + \alpha_y^2$$

where, referring to Fig. 2

- $\alpha_y$  decay factor of the evanescent wave in the  $y$  direction in the air-filled region,
- $\beta_y$  wavenumber in the  $y$  direction in the dielectric area,
- $\beta$  wavenumber in the propagation ( $z$ ) direction, equal to  $\beta_e$  for the even modes and  $\beta_0$  for the odd modes,
- $\beta_z$  wavenumber of the uncoupled guide,
- $\epsilon_r$  dielectric constant of guide track,
- $a$  plate separation,
- $b$  dielectric height,
- $d$  gap between coupled guides.

Fig. 3 compares typical transmission responses obtained through calculation by means of the above equations and results obtained by measurement for this section. The measured resonant coupled lengths correspond quite well to the calculated values, but the gap widths necessary to obtain the same response as for the theoretical case can differ from the theoretical by as much as 30 percent.

The differences between the two curves can be ascribed to the fact that the short circuits are not true short circuits, as the metal strips extend partly into the gap between the two guides and obviously disturb the fields in the immediate vicinity of the main guide. This effect is not taken into consideration in the theoretical calculation. If the shorts are made to extend for a shorter distance into the coupled region, the unwanted end-coupling effect is noticed again.

The reason why the resonant frequency is so close to the calculated value can be found in the fact that the algebraic mean of the even- and odd-mode wavenumbers  $\beta_c$  is very close to the wavenumber of the uncoupled guide  $\beta_z$  and that this is not strongly affected by the port terminations. For instance, for a coupled line section that resonates at

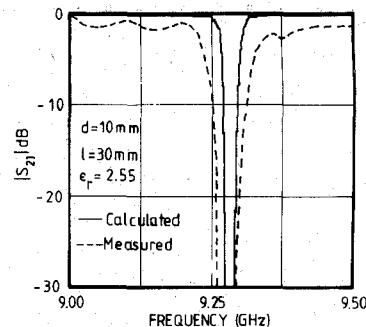


Fig. 3. Calculated and measured response of coupled section.

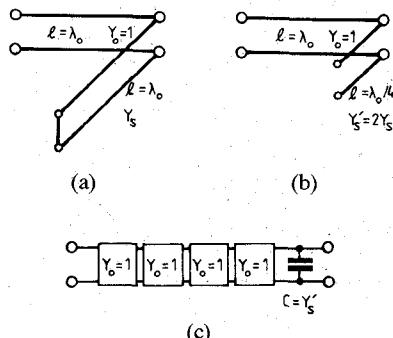


Fig. 4. Equivalent circuits. (a) Full-wave shunt stub. (b) Quarter-wave shunt stub for narrow-band equivalence to circuit at (a). (c) Richards's equivalent circuit.

9.57 GHz, with a 15-mm gap and 50-mm coupled length,  $\beta_c = 122.10 \text{ rad/m}$ , while  $\beta_z = 121.99 \text{ rad/m}$  at 9.5 GHz.

Clearly, these two wavenumbers are, to a good approximation, equal for cases of loose coupling that are relevant for narrow-band filters.

### III. EQUIVALENT CIRCUIT OF TWO COUPLED GUIDES

The phase of the transmission coefficient of the coupled guide section can readily be found from (1) as

$$= \arg S_{21} = -\beta_c l + \tan^{-1} \left[ \frac{\sin^2 Cl \sin^2 \beta_c l}{(1 + \cos^2 Cl)(1 - \cos 2\beta_c l)} \right]. \quad (7)$$

After some trigonometrical manipulation, this reduces to

$$= -\beta_c l + \tan^{-1} \left[ 2 \frac{1 - \cos^2 Cl}{1 + \cos^2 Cl} \cot \beta_c l \right]. \quad (8)$$

Consider next the cascade of a unit element of length  $l_u$  and shunt short-circuited stub of length  $l$  with characteristic admittance  $Y_s$ , as shown in Fig. 4(a). The stub has an input admittance

$$Y = -Y_s \cot(\beta_z l) \quad (9)$$

while the phase of the transmission coefficient of the section is given by

$$= \arg S_{21} = -\beta_z l_u + \tan^{-1} (Y_s / 2 \cot \beta_z l). \quad (10)$$

Comparison of (8) and (10) shows that the coupled guide section can be modeled as a shunt stub and uncoupled line if certain conditions are met. Firstly, the propagation constants and line lengths must correspond, which is approximately true. Secondly, the admittance of the stub is given by (13), the implication of which will be discussed shortly.

In order to achieve sufficient coupling for lines that are not too closely spaced, the line length must be increased. However, the increase in length must be in steps of  $\lambda_0/2$  so that the resonance conditions can be maintained. In practice, it was found that lines of a full wavelength gave sufficient coupling and the correct resonant frequency.

Now

$$\cot(2\pi + \theta) = -\tan(\pi/2 \pm \theta) \quad (11)$$

so that, if the length of the stub, now open-circuited, is made one-quarter wavelength ( $l_s = l/4$ ) the phase of the equivalent section is given by

$$= -\beta_z l_u - \tan^{-1}(Y_s/2 \tan \beta_z l_s). \quad (12)$$

By assuming that  $\beta_z = \beta_c$ , the phases described in (8) and (12) are equal if

$$Y_s = 4 \frac{1 - \cos^2 Cl}{1 + \cos^2 Cl} \quad (13)$$

or

$$Cl = \cos^{-1} \sqrt{\frac{4 - Y_s}{4 + Y_s}}. \quad (14)$$

This means that the equivalent of the coupled guide section can be found in the form of a length of *uncoupled* guide cascaded with an open-circuited stub shunt connected across the guide in a point, as shown in Fig. 4(b). The coupling factor  $C$ , however, is a function of the frequency, so that a varying impedance is implied. This variation is fairly slow, and the value of  $Cl$  is small, so that the variation in  $Y_s$  is acceptable, provided once again the filter bandwidth is small.

Another approach is to compare the moduli of the two transfer functions. For the coupled line section, we obtain

$$|S_{21}|^2 = \frac{4}{4 + \frac{(1 - \cos^2 Cl)^2}{\sin^2 \beta_c l \cos^2 Cl}} \quad (15)$$

while for the equivalent circuit

$$|S_{21}|^2 = \frac{4}{4 + Y_s^2 \cot^2 \beta_z l}. \quad (16)$$

Comparison shows that

$$Y_s \cot \beta_z l = \frac{1 - \cos^2 Cl}{\sin \beta_c l \cos Cl}. \quad (17)$$

The two sides of (17) cannot everywhere be equal; however, they differ at the lower frequencies as cot versus csc, where their contribution to the transfer function is negligi-

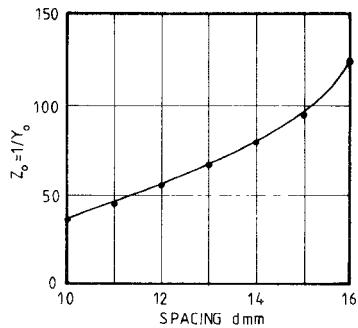


Fig. 5 Variation of normalized characteristic impedance of the equivalent shunt stub with gap.

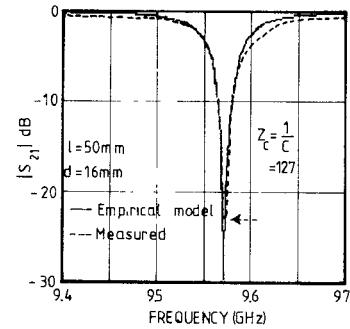


Fig. 6. Comparison of measured response of the coupled line section and the equivalent circuit.

ble. At the higher frequencies, where both sides are approaching the singularity at  $360^\circ$  very rapidly, the differences are small and immaterial.

Under Richards's transform [4], the section has the equivalent of a shunt capacitor cascaded with four unit elements, the length of which is well in excess of  $\lambda/4$ , as shown in Fig. 4(c); the way in which this is accommodated in the design is discussed below. See also [5] for a description of filter design using Richards's transform.

#### IV. EMPIRICAL EQUIVALENT CIRCUIT

The equivalent circuit described above unfortunately does not fit the practical circuit, due to the contributions of the metal short-circuiting plate. However, the latter are situated one wavelength apart, so that the electrical equivalent of any one of them may be moved to the position of the other, at one end of the circuit. As it has already been established that the equivalent of the network is a Richards capacitor, the reactive contribution from the short may be added to the equivalent network, and since it is obviously lumped, it only affect the value of the shunt capacitor will to some extent, will have little effect on the frequency response, and will have virtually no effect on the resonant frequency.

The simplest way of taking into consideration the contribution from the shorts is simply absorbing it into the equivalent circuit. The transmission response of a coupled line circuit is measured, and a value of admittance is fitted to the equivalent circuit such that the same frequency response is obtained. This gives a Richards equivalent

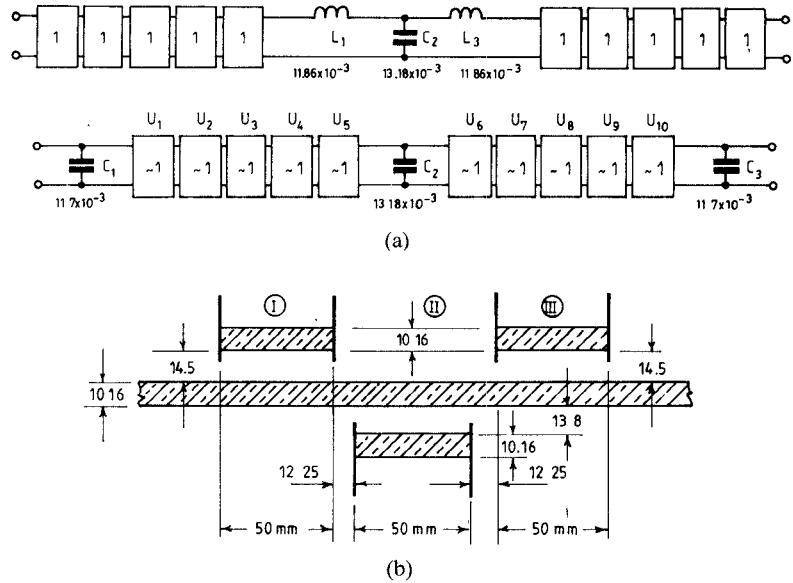


Fig. 7. (a) Development of a distributed prototype from a lumped-element filter (b) Realization in NRD.

circuit similar to that of Fig. 4(c), and this was done for a number of coupled line sections. A curve was fitted to the result.

Fig. 5 shows the variation of characteristic impedance of the equivalent stub versus the gap of the coupled guide, while Fig. 6 shows the agreement between the measured amplitude response and that of the empirical shunt capacitor model for a shunt stub of a normalized admittance of  $7.87 \times 10^{-3}$  (a normalized impedance of 127).

## V. FILTER DESIGN AND MEASUREMENTS

The filter design follows the conventional procedure [5] for a bandstop structure. In this case, a third-order Chebyshev prototype was used. The physical length of the coupled guide section is, however, one wavelength long instead of the quarter wavelength normally used in filter design. Consequently, five unit elements are transformed into the circuit from each port end by means of the Kuroda transform [5]. This affords sufficient length to fit in the coupled guide sections physically. After transformation of the unit elements across the end sections as shown in Fig. 7(a), the filter was realized as three coupled guides as shown in Fig. 7(b).

Referring to Fig. 7(b), the capacitor  $C_1$  and the unit elements  $U_1$  to  $U_4$  were realized as the coupled line section denoted as I. The remaining unit element,  $U_5$ , spaces the coupled line sections I and II, while  $C_2$  and  $U_6$  to  $U_9$  are realized as section II. Once again,  $U_{10}$  spaces sections II and III.  $C_3$  is realized by using four unit elements from the right port as section III.

Due to the very narrow relative bandwidth, the unit-element impedance levels are only very slightly changed by each Kuroda transform, and in practice the impedance level was taken as constant and equal to unity.

The filter was constructed using Polypenco Q200.5 cross-linked polystyrene as a dielectric, with  $\epsilon_r = 2.55$ , a

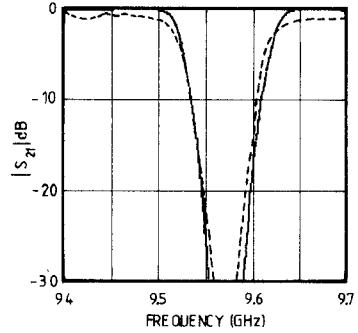


Fig. 8. Transmission response of trial filter.

height of 10.16 mm, and a plate separation of 15 mm. The structure was connected to an X-band network analyzer through the pair of transitions described in [6]. (X-band is used for modeling because of the costs of machining involved at the millimeter-wave frequencies.)

Fig. 8 shows the measured transmission response for the filter compared to the predicted response based on the transmission-line (Richards) equivalent. This equivalent circuit predicts a frequency response with spurious responses centered at 28.71 GHz and other odd multiples of 9.57 GHz, the center frequency of the filter. In reality, of course, the element lengths are a full wavelength, so that two subharmonic responses, at 0 and 4.79 GHz, should exist, together with the harmonic responses at multiples of 4.79 GHz, i.e., 9.57 GHz, 14.36 GHz, etc. Furthermore, the network consists of waveguide sections, so that the whole of the frequency region to zero frequency is mapped into the region above the cutoff frequency, 8.55 GHz in this case. In practice, the response at the cutoff frequency disappears in the cutoff rolloff, but a spurious response is noted at 8.75 GHz, which corresponds exactly to the mapped subharmonic response. The predicted spurious response at 14.36 GHz was not noted because the network

analyzer only measured to 12 GHz. Beyond 10 GHz, the frequency at which the guide ceases to support NRD-modes, the transmission response does, however, deteriorate, indicating radiation from between the plates.

## VI. CONCLUSIONS

The frequency response of the trial filter agrees well with the design value, in spite of the approximations that have to be made to model the coupled guides. This indicates that the mechanisms that control the performance of the network have been accurately described over the bandwidth of interest. It is of interest at this point to note that the specific dimensions and materials used, using a standard waveguide height and plate separation of 15 mm, give a useful band with NRD-properties of 8.55 to 10 GHz, which is rather limited. If the plate separation is made 11.43 mm, to correspond with the maximum frequency of standard X-band, and if a dielectric with  $\epsilon_r = 10$  is used, the fundamental mode cutoff frequency is found to be 5.99 GHz, which is slightly below the 6.56 GHz for the standard guide.

In practice, the higher waveguide bands, centered around 35, 60, and 95 GHz, would of course be used, where the proposed structure remains the only bandstop filter in NRD. It is envisaged that in the future an equivalent for the coupled lines without short circuits can be developed, possibly leading to tighter coupling of the lines and, consequently, shorter filters.

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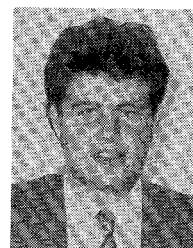
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